Numerical calculation methods for neutron noise in heterogeneous reactor cores

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1. Introduction

- Neutron noise = the difference between the time-dependent flux and its mean value, provides interesting dynamic information of the core status and can be measured by in-core and ex-core detectors.
- The noise technique can be used online without disruption of the reactor operation.
- Possibility to investigate the neutron noise behaviour and interpret the measurement.
- Support the instrumentation development.
- Numerical simulation remains a challenge for describing detector signals and improving core surveillance.

2. Noise equation in multi-group diffusion theory

- To solve the neutron noise equation, it is necessary to define a noise source through the fluctuations of static cross sections, and the static state solutions of the k_{eff} and fluxes. Thus, the solution of the static equation is required.
- The static equation in multi-group diffusion theory:

$$-\nabla \cdot [D_g \nabla \phi_g(\mathbf{r})] + \Sigma_{t,g}\phi_g(\mathbf{r}) = \frac{1}{k_{eff}}\chi_g \sum_{g'} \nu\Sigma_{f,g'}\phi_{g'}(\mathbf{r}) + \sum_{g' \neq g} \Sigma_{s,g' \to g}\phi_{g'}(\mathbf{r}) \tag{1}$$

where,

$$\chi_g = (1-\beta)\chi_g^p + \sum_j \beta_j \chi_{g,j}^d. \tag{2}$$

All symbols have their usual meaning.

2. Noise equation in multi-group diffusion theory

The multi-group noise equation is derived from the space- and time-dependent diffusion equations by assuming that all time-dependent terms, $X(\mathbf{r}, t)$, can be split into a stationary component plus a small fluctuation as

$$X(\mathbf{r}, t) = X_0(\mathbf{r}) + \delta X(\mathbf{r}, t), \tag{3}$$

By assuming that the fluctuations are small so that only the first order noise needs to be taken into account. Performing a Fourier transform of time-dependent terms, $\delta X(\mathbf{r}, t)$, as

$$\delta X(\mathbf{r}, \omega) = \int_{-\infty}^{\infty} \delta X(\mathbf{r}, t) e^{-i\omega t} dt. \tag{4}$$

2. Noise equation in multigroup diffusion theory

The first order neutron noise in multi-group diffusion theory is:

$$-\nabla\cdot\left[D_g \nabla\delta\phi_g\left(\mathbf{r},\omega\right)\right]+\Sigma_{t,g}\left(\omega\right)\delta\phi_g\left(\mathbf{r},\omega\right)=\frac{1}{k_{eff}}\chi_g\left(\omega\right)\sum_{g'}\nu\Sigma_{f,g'}\delta\phi_{g'}\left(\mathbf{r},\omega\right)$$

$$+\sum_{g'\neq g}\Sigma_{s,g'\to g}\delta\phi_{g'}\left(\mathbf{r},\omega\right)+S_g\left(\mathbf{r},\omega\right), \qquad (5)$$

where, $\delta\phi_g\left(\mathbf{r},\omega\right)$ is the noise in group g.
The frequency-dependent total cross section is

$$\Sigma_{t,g}\left(\omega\right)=\Sigma_{a,g}\left(\omega\right)+\sum_{g'\neq g}\Sigma_{s,g\to g'}, \qquad (6)$$

$$\Sigma_{a,g}\left(\omega\right)=\Sigma_{a,g}+\frac{i\omega}{\upsilon_g}. \qquad (7)$$

$\chi_g(\omega)$ is the frequency-dependent fission energy spectrum:

$$\chi_g\left(\omega\right)=\chi_g-\sum_j\chi^d_{g,j}\frac{i\omega\beta_j}{\lambda_j+i\omega}. \qquad (8)$$

2. Noise equation in multigroup diffusion theory

The noise source is calculated from the fluctuations of macroscopic cross sections

$$S_g(\mathbf{r},\omega) = -\delta\Sigma_{a,g}(\omega)\,\phi_g(\mathbf{r}) - \sum_{g'\neq g}\delta\Sigma_{s,g\to g'}(\omega)\phi_g(\mathbf{r}) + \sum_{g'\neq g}\delta\Sigma_{s,g'\to g}(\omega)\phi_{g'}$$

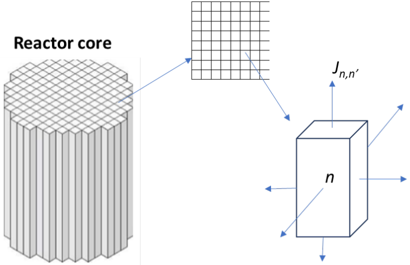
$$+\frac{1}{k_{eff}}\chi_g(\omega)\sum_{g'}\delta\left[\nu\Sigma_{f,g'}(\omega)\right]\phi_{g'}(\mathbf{r}). \quad (9)$$

3. Spatial discretization

The balance (static or noise) equation in each group in a node n is

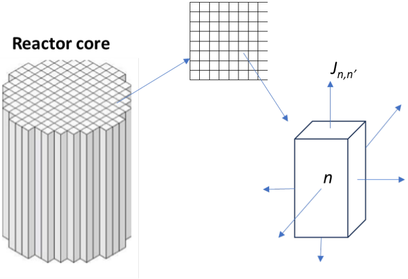
$$\sum_{n'=1}^{N} A_{n,n'} J_{n,n'} + \Sigma_t \varphi_n V = Q_n V \tag{10}$$

$J_{n,n'}$: the surface-averaged net current from node n through the interface with a neighboring node n',
$A_{n,n'}$: the area of the interface of the two nodes,
φ_n: the static flux or the noise,
Q_n: the total source.



3. Spatial discretization

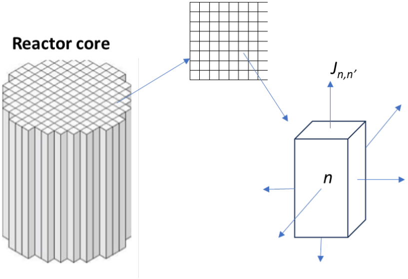
1. **Nodal methods**
   - **Advantages:** can handle a large node size with correction of the net current.
   - **Disadvantages:** not applicable in the cases of point-like sources and/or vibration.



3. Spatial discretization

1 Finite difference (FD) approximation

- **Advantages:** simple and flexible with node size, suitable to the cases of point-like sources and/or vibration.
- **Disadvantages:** problem size may be large, slow conversion speed for the cases of point-like sources and/or vibration



Spatial discretization

1. **Finite difference (FD) approximation**

The net current $J_{n,n'}$ is calculated as

$$J_{n,n'} = a_{n,n'} \left(\varphi_n - \varphi_{n'} \right). \tag{11}$$

The coupling coefficient $a_{n,n'}$ is

$$a_{n,n'} = \frac{2}{h} \frac{D_n D'_n}{D_n + D'_n}, \tag{12}$$

If the neighboring node n' is a boundary, the current is

$$J_{n,n'} = \frac{(1-\alpha) 2 D_n}{h + 4 D_n} \varphi_n \tag{13}$$

where, a vacuum boundary has $\alpha = 0$ and a reflective boundary has $\alpha = 1$.

4. Numerical methods

Depending on the size of problem ($N \times G$) to chose appropriate solution methods.

1. **Green/Transfer function**

$$\delta\phi(\mathbf{r}, \omega) = [G_{XS}(\mathbf{r}, \mathbf{r'}, \omega) \times \phi(\mathbf{r'}]\delta_{XS}(\mathbf{r'}, \omega) \tag{14}$$

- **Advantages:** Transfer function is solved only one, and applicable to any case of noise calculation with any source type.
- **Disadvantages:** time and memory consuming, not applicable to a large case with multi-energy groups

4. Numerical methods

- **Finite difference (FD) approximation**
- Comparison with analytic solution in case of point-like source

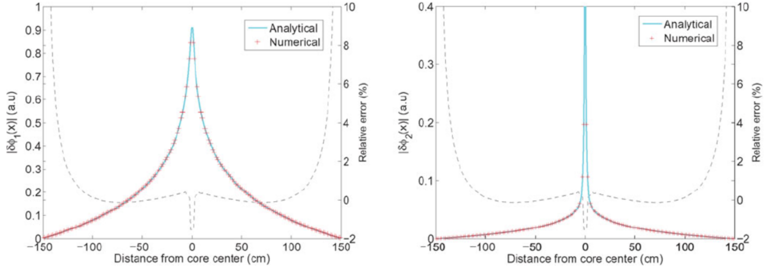


Figure 1: Comparison of fast and thermal noise in a PWR between numerical and analytic calculations.

5. Neutron noise calculation in a PWR

1. 2D, two-groups model
2. Finite difference for spatial discretization
3. Finer meshes for vibrating assembly
4. Transfer function

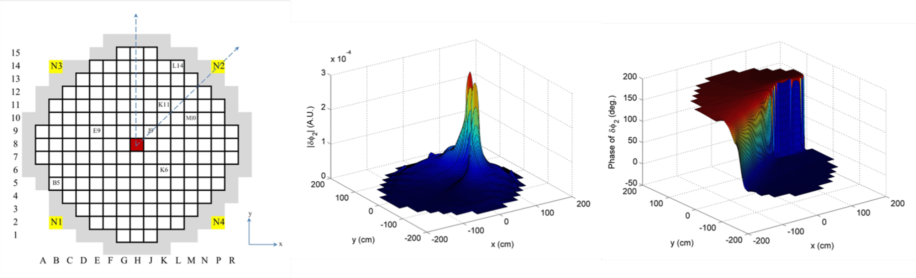


Figure 2: Magnitude and phase of thermal noise induced by vibration of assembly M10 in x-direction ($f = 8Hz$).

5. Neutron noise calculation in a PWR

1. Space-dependent noise induced by fuel assembly vibration

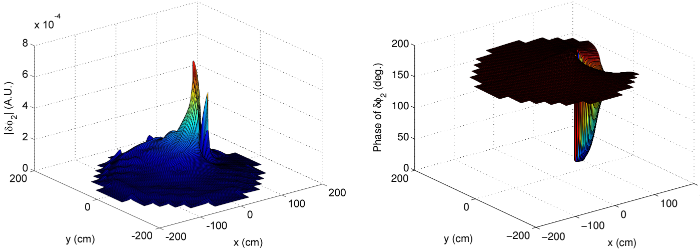


Figure 3: Magnitude and phase of thermal noise induced by trajectory vibration of assembly M10 in xy-direction ($f = 8$ Hz).

5. Neutron noise calculation in a PWR

1. Ex-core noise induced by fuel assembly vibration

- The increase of noise amplitude with burnup is only found for a subset of fuel assemblies located at the periphery.

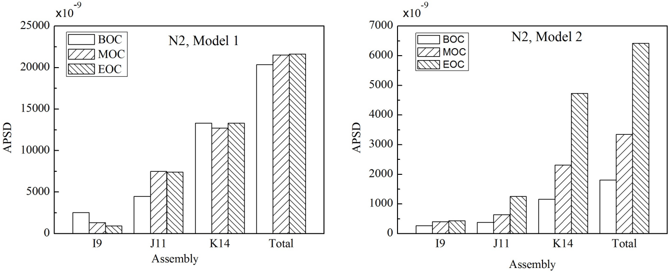


Figure 4: APSD (a.u.) of ex-core detector induced by simultaneous stochastic vibrations of fuel assemblies J9, K11 and L14 in a PWR.

5. Neutron noise calculation in a PWR

1. Ex-core noise induced by fuel assembly vibration

- The noise induced by the peripheral assemblies dominates the ex-core detector signals.

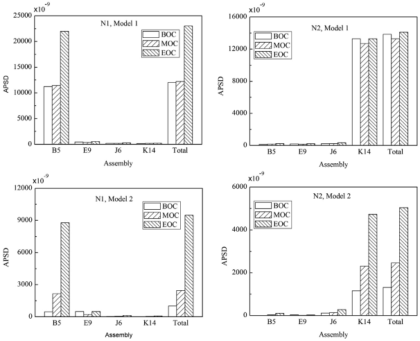
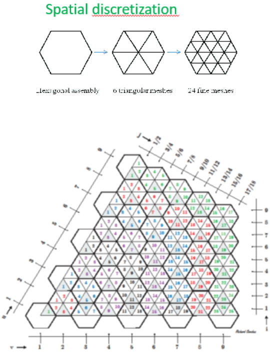


Figure 5: APSD (a.u.) of detectors N1 and N2 induced by simultaneous stochastic vibrations of assemblies B5, E9, K6 and L14 in a PWR.

6. Neutron noise calculation for fast reactors

1. Two modules: static and noise
2. Multigroups, several delayed neutron precursors
3. Diffusion theory, hexagonal geometry, finite differences, power iteration
4. CMFD acceleration



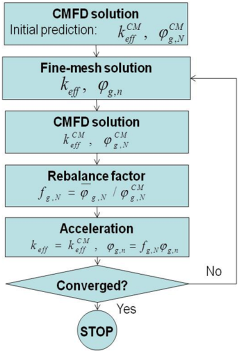


Figure 6: Triangular discretization and flowchart of noise solution for hexagonal

6. Neutron noise calculation for fast reactors

1. SFR core
2. 33 energy groups, 6 delayed neutron precursors
3. Noise source: perturbation of coolant density at the core central

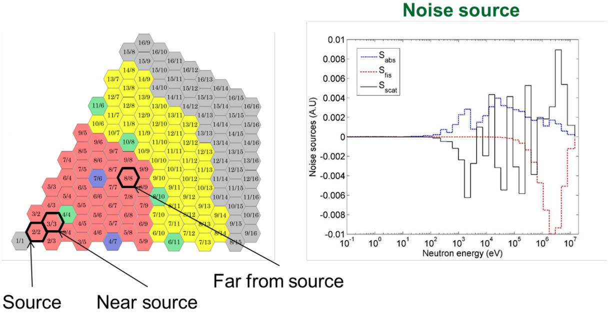


Figure 7: Triangular discretization and flowchart of noise solution for hexagonal reactor.

6. Neutron noise calculation for fast reactors

1. Energy-dependent noise induced by perturbation of coolant density at the core central.

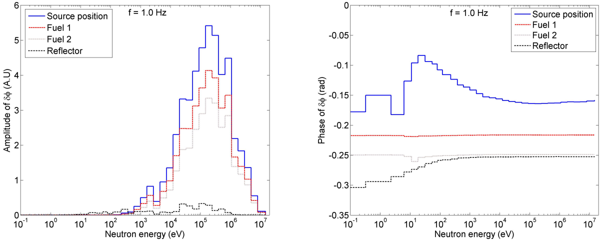


Figure 8: Amplitude and phase of noise at different locations in the core: Source position, near source, far from source and Reflector ($f = 1.0$ Hz).

6. Neutron noise calculation for fast reactors

1. Space-dependent noise induced by perturbation of coolant density at the core central.

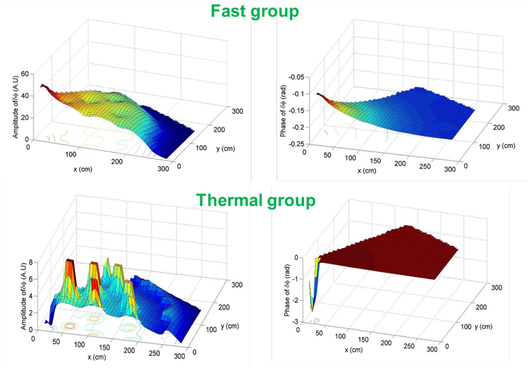


Figure 9: Amplitude and phase of space-dependent noise in fast (group 9) and epi-thermal (group 20) groups ($f = 1Hz$).

Thanks for your attention!

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