Mach Stem Height Estimation in Strong Reflection Domain for Overexpanded Jets

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ABSTRACT The investigation of Mach reflection (MR) configuration in steady supersonic flows has received considerable attention in recent decades due to its widespread occurrence in a diverse range of various aerospace, defence, and industrial applications. Despite the progress in understanding the Mach reflection configuration, some significant gaps and ambiguities persist in the MR configuration, especially in the strong Mach reflection domain (MR configuration with subsonic flow behind the reflected shockwave). From Azevedo and Liu [1] to Bai and Wu [2], the numerous analytical works on Mach stem height concentrate primarily on Mach stem height and Mach reflection configuration in the weak reflection domain (i.e. flow is supersonic behind the reflected shockwave). Except for the work of Qin et al [5], there are currently no analytical studies available that can accurately predict the height of the Mach stem or the configuration of the Mach reflection in the strong reflection domain. Similarly, the works of Li and Ben-Dor [3] and Paramanantham et al [4] extended the above analytical works on the wedge flows to overexpanded jets. Nevertheless, the analytical models for the overexpanded jets also suffered from the same shortcomings that plagued the previous analytical models i.e. the inability to account for subsonic flow occurring behind the reflected shock wave. In this study, we aim to extend the analytical model developed by Qin et al [5] by applying it to the overexpanded jet (to the models developed by Paramanantham et al [4]). The objective is to estimate the height of the Mach stem in the domain of strong Mach reflection. This will also function as a significant metric to evaluate the reliability and accuracy of the Qin et al [5] model. The analytical models will be validated using higher-order shock-capturing numerical simulations.

1 INTRODUCTION

The problem of shock reflection and its interaction gained prominence with the development of spacecrafts and missiles capable of flying at supersonic speeds i.e. flying at a few times the speed of sound. In supersonic flows, shockwaves and expansion fans are inevitable phenomena that occur either to match the pressure or to satisfy the flow-turning condition. The interaction of these waves, particularly the shockwaves, can produce interesting flow configurations and has been extensively studied in steady, pseudo-steady and unsteady flows. Despite the advances in our understanding of shock reflection phenomena, there are regimes in which the shock reflection configuration or modelling of the reflection configuration is either absent or poorly understood. One of the notable unexplained phenomena in the shock reflection problem is the estimation of Mach stem height in the strong reflection domain, in which the flow behind the reflected shock wave is subsonic. The modelling of the Mach stem height in the strong reflection domain is impeded due to the following issues. First, at transonic Mach numbers, the three-shock theory fails to predict solutions to Mach reflection configurations for specific combinations of Mach numbers and flow deflection angle, resulting in the well-known von Neumann paradox. Second, the subsonic region behind the reflected shock wave is difficult to quantify using any known analytical method. Because of these difficulties, the Mach stem height in steady flows was not modelled. Qin et al. [5] recently developed an analytical model for predicting Mach stem height in the strong shock reflection regime. Their model was primarily applied to wedge flows and predicted with reasonable accuracy the Mach stem height in the flow field. In this study, we extend the Qin et al [5] model for estimating Mach stem height in the strong shock reflection domain to overexpanded jets. The current study attempts to fill the modelling gap that is present in the analytical models for Mach stem height estimation in the overexpanded jet, as discussed in the paper by Paramanantham et al. [4]

2 METHODOLOGY

The schematic of the overexpanded jet in the strong reflection domain is shown in Figure 1. The analytical model involves solving the three-shock theory for the given nozzle pressure ratio and nozzle exit Mach number. After obtaining the flow variables at the triple point, it is necessary to model the flow across the reflected shock and the expansion fans. The absence of interaction between the expansion fan and the reflected shockwave simplifies the modelling of these waves in overexpanded jets. Once the flow across the reflected shockwave and expansion fan are solved, the Mach stem height can be estimated by appropriate geometrical relations.

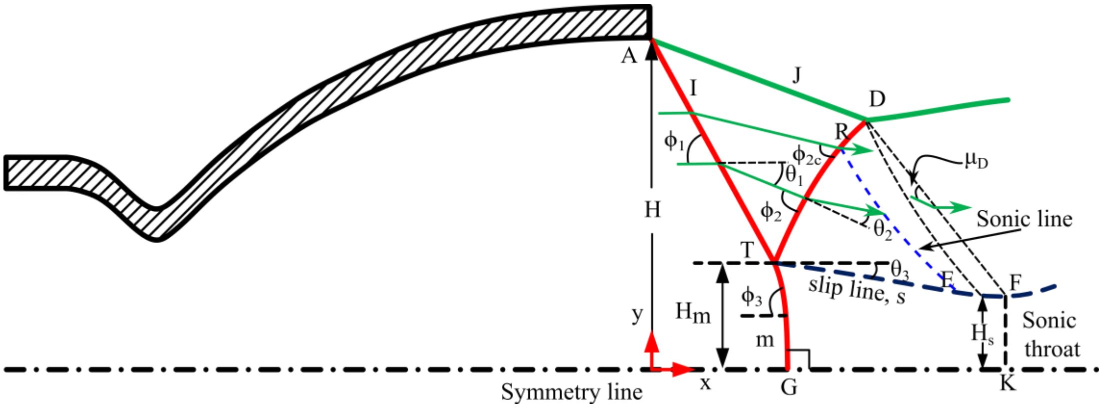

Figure 1. Schematic of the Mach reflection configuration for an overexpanded jet in strong shock reflection domain.

The reflected shockwave is modelled in the same manner as that of Qin et al. [5]. The curvature of the reflected shockwave is expressed using the second-order Taylor series approximation as given in equation 1.

$$y_D - y_T = \frac{\tan(\phi_2 - \theta_1)\tan(\phi_{2c} - \theta_1)}{\tan(\phi_2 - \theta_1) + \tan(\phi_{2c} - \theta_1)}\,(x_D - x_T) \tag{1}$$

Where ϕ *and* θ represents the shock angle and flow deflection angle respectively. Subscripts 1 and 2 refer to the flow behind the incident shockwave and reflected shock wave respectively. And the subscript 'c' refers to the sonic condition. The curved expansion fan is modelled using an average Mach angle obtained at the location D and F respectively and given in equations 2 and 3.

$$y_D - y_F = \tan\left(\frac{\mu_D{}'}{2}\right)(x_D - x_F) \tag{2}$$

$$\mu_D{}' = \mu_D + \mu_F \tag{3}$$

The above equations, combined with the equations for the incident shockwave, slipline, Mach stem, and jet boundary given by Paramanantham et al [4], form a set of seven equations that can be solved to determine the Mach stem height. To validate the analytical model, these results were compared to numerical simulations performed with the higher-order WENO scheme.

Table 1. Analytical and numerical solution of Mach stem height for an overexpanded jet of M = 1.5 for different NPR.

S.NO	**Mach Number (M)**	**Nozzle Pressure Ratio (NPR)**	**Analytical Solution** $(\frac{H_m}{H})$	**Numerical Solution** $(\frac{H_m}{H})$
1	1.5	2.1	0.362	0.401
2	1.5	2.2	0.333	0.343
3	1.5	2.3	0.294	0.273
4	1.5	2.4	0.263	0.170

3 RESULTS AND DISCUSSION

The analytical model developed for the strong reflection domain is used to determine the Mach stem height for a nozzle exit Mach number of 1.5. The analytical results for nozzle pressure ratios ranging from NPR = 2.0 to 2.7 are presented in Table 1. The analytical model accurately reflects the trend of decreasing Mach stem height with increasing NPR. As the NPR rises, the Mach reflection moves downstream of the nozzle exit and becomes regular reflection at NPR = 2.7 for M = 1.5. Table 1 compares the numerical results of the Mach stem obtained from the numerical schlieren with the analytical results. We can see that the analytical model performs well at lower NPR, i.e. under highly overexpanded jet conditions, while there is a discrepancy at higher NPR. The present model also fails to properly predict the detachment condition. The reason for the discrepancy in the analytical model to capture the detachment condition is not known and will be investigated. A thorough comparison of the current model with the numerical solutions for different Mach number jets will be performed, and the results will be presented.

4 REFERENCES

1. Azevedo, D. J., & Liu, C. S. (1993). Engineering approach to the prediction of shock patterns in bounded high-speed flows. *AIAA Journal*, *31*(1), 83–90. https://doi.org/10.2514/3.11322.

2. Bai, C.-Y., & Wu, Z.-N. (2017). Size and shape of shock waves and slipline for Mach reflection in steady flow. *Journal of Fluid Mechanics*, *818*, 116–140. https://doi.org/10.1017/jfm.2017.139.
3. Li, H., & Ben-Dor, G. (1997). Mach Reflection wave configuration in Two-Dimensional Supersonic Jets of Overexpanded Nozzles. *AIAA Journal*, *36*(3), 488–491.
4. Paramanantham, V., Janakiram, S., & Gopalapillai, R (2022). Prediction of Mach stem height in compressible open jets. Part 1 Overexpanded jets. *Journal of Fluid Mechanics*, *942*, A48. https://doi:10.1017/jfm.2022.374.
5. Qin, Z., Shi, A., Dowell, E. H., Pei, Y., & Huang, E. (2022). Analytical Model of Strong Mach Reflection. *AIAA Journal*, *60*(9), 5187–5202. https://doi.org/10.2514/1.J061701.