Prediction of Pseudo Steady Shock Refraction Over Air-Water Interface Using Fourier Neural Operator

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ABSTRACT Fourier neural operator is used to predict the complex refraction of moving shock over air water interface for different shock Mach numbers (M_s) of 3.4 to 4.4 and inclination angle (β) ranging from 35° to 85°.

1 INTRODUCTION

The interaction of shock waves with an interface separating fluids of different densities can result in a wide variety of hydrodynamic motion. Underwater explosions near heavy structures such as gravity dams, wind turbines, and ships create a strong shock or detonation wave propagating outwards interacting with the interface. Understanding these interactions gives insight into shock-induced loading and helps mitigate damages. When the shockwave comes in contact with the interface during the initial times, the refraction phenomena dominate the problem. The class of studies dealing with such pseudo-steady phenomena is called the shock-refraction problems. Shock refraction can be studied in two types of interfaces: slow-fast and fast-slow. If the speed of sound of the first medium where the shock originates is slower than the second medium, the interface is called slow-fast and vice-versa.

Over the years, shock refraction has been studied thoroughly in gas-gas interfaces [Abd-El-Fattah and Henderson (1976); Polachek and Seeger (1951)]. There are two types of refraction patterns: regular and irregular refractions. A regular refraction has three waves: incident, reflected and transmitted, all intersecting at a single point on the interface. Irregular refraction comprises of a complex arrangement of more than three waves. The refraction pattern is determined by the strength of the incident shock wave (M_s) and the inclination angle (β) the interface makes with the shock wave. The strength of the shock wave also determines the refraction sequence with increasing inclination angle. Based on the refraction sequence, different regimes have been identified in the gas-gas interfaces, as shown in Table 1, compiled by Nourgaliev et al. (2005)

Slow-Fast interface	
Incident shock strength	***Refraction pattern with increasing β***
Very weak	RRE→RRR→BPR→FPR→FNR
Weak	RRE→RRR→BPR→FNR→TRR→TNR
Strong	RRE→BPR→TMR
Fast-Slow interface	
Very weak	RRR→RRE→CFR (ARE)
Weak, Strong, Stronger	RRR→MRR→CFR

Table 1: Different refraction patterns for different conditions of shock strength, inclination angle and acoustic speed of the medium

List of abbreviations and its definitions:

RRE: Regular Refraction with reflected Expansion wave
TRR: Twin Regular Refraction
TMR: Twin Mach Refraction
BPR: Bound Precursor Refraction
TNR: Twin von Neumann Refraction
MRR: Mach reflexion type Refraction
FPR: Free Precursor Refraction
RRR: Regular Refraction with Reflected shock
ARE: Anomalous Refraction with reflected Expansion wave

FNR: Free Precursor von Neumann Refraction

CFR: Concave-Forwards irregular Refraction

In multiphase interfaces such as air-water (slow-fast), the refraction patterns were not identified until very recently. Anbu Serene Raj C et al. (2024) performed a comprehensive study, including experiments and numerical simulations, to identify the refraction sequence and transition criteria in the very weak incident shock strength regime. They captured refraction patterns that were previously not seen in air-water interfaces such as RRR and BPR. The predicted transition criteria from their analytical study helped identify the shock refraction sequence in weak and strong regimes of an air-water interface as shown in Table 2.

Slow-Fast (gas-liquid) interface	
Incident shock strength	***Refraction pattern with increasing β***
Very weak	RRR→BPR→FPR→FNR
Weak	RRR → IRMR → FMR
Strong	RRR → IRMR

Table 2: Different refraction patterns for different inclination angle and acoustic speed of the medium for very weak incident shock regime.

Generally, multiphase computations of higher order of accuracy in both space and time is required to compute the flow fields of moving shock wave/ pseudo steady shock wave over air-water interface. However, the computations require a highly refined mesh and enormous amount of time to compute the flow field. This introduces high cost which cannot be affordable for industries or small-scale research institutes. The current paper aims to use Fourier neural network, a machine learning framework to model the complex air-water shock interaction which can provide faster results with less cost and time.
Recently, there has been tremendous growth in machine learning due to the advent of generative pre-trained transformers like ChatGPT and Gemini. The universal function approximation theorem [Cybenko (1989)] and universal operator approximation theorem [Chen & Chen (1995)] helped the researchers to apply machine learning to the non-linear fluid dynamics problems. The operator network is similar to the standard neural network in which linear transformations and nonlinear activation functions are replaced by the linear operators and non-linear activation operators. The Fourier neural operator [Li et al. (2020)] can learn mapping or operator from one functional space to another by optimizing the weights of the network. The Fourier neural operator takes a set of varied initial condition as input and outputs the flow field after a stipulated time via a sequence of Fourier operations. The output predicted by the operator network is compared with the known flow field obtained either computationally/ experimentally and calculate the loss function which in turn help to optimize the network using back propagation.

In the current paper, Fourier operator network is applied for the first time to air-water shock refraction to learn an operator which can predict the refraction pattern for a given initial shock speed and the inclination angle.

2 METHODOLOGY

The Euler equations are solved computationally using a finite volume second order solver BlastForm for the incident shock Mach numbers (M_s) ranging from 3.4 to 4.4 and inclination angle (β) ranging from 35° to 85°. The initial condition and the flow patten at a time of 1.6e-5 is extracted from the computational results and curated a dataset for training and testing the operator network. The input data is raised to a high dimensional one using linear layers and then passed to four Fourier layers, and eventually taken back to the output space via another linear layer. Each Fourier layer consist of a fast Fourier transform for filtering the high dimensional data for a definite number of modes and then an inverse Fourier transform is performed on the filtered data. The high dimensional data is also applied with a linear transformation and then combines with the inverse Fourier transform and finally non linearity is introduced using non-linear operator Gelu. The optimizer used for the gradient descent is Adam with learning rate of 1e-3 and weight decay of 1e-4. The network is trained on Nvidia RTX 4000 ADA GPU.

3 RESULTS AND DISCUSSION

The Fourier neural network is trained on the dataset generated from the computations using the BlastForm for moving shock Mach numbers of 2.2, 3.4 and 4.4 and inclination angle ranging from 30° to 85°. The contour of the gradient for log(pressure)*log(density) of initial condition and flow pattern after 1.6e-5 is given in the figure 1 (a) and (b), respectively for $M_s = 4.4$, $\beta = 70°$.

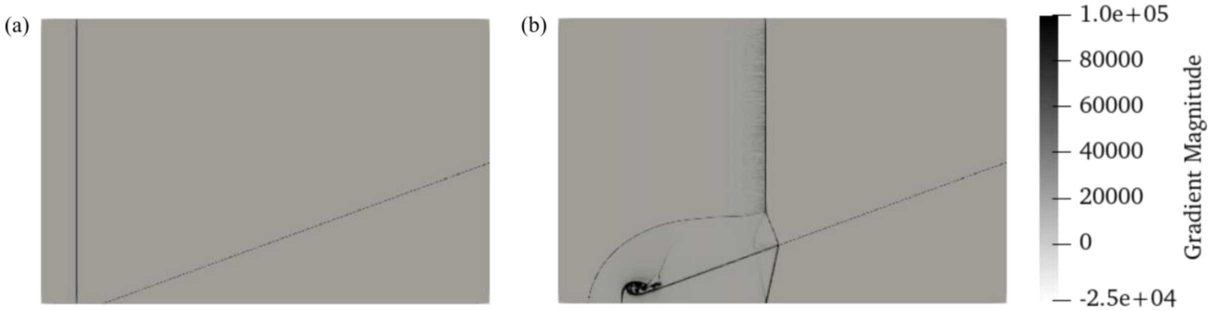

Figure 1: The contour of the gradient for log(pressure)*log(density) for $M_s = 4.4$, $\beta = 70°$ (a) Initial condition: Shock wave originating in air (left) moving to the right (inclined water interface) (b) flow field after 1.6e-5 s displaying an IRMR with a TMR in air and transmitted oblique shock wave inside water.

Since the data curation of all possible refraction took long time, the Fourier neural network is tested for one dimensional Burger's equation which is a model equation for Eulers equation. The dataset is created by solving the PDE using conventional method using FFT followed by a Runge-Kutta solver and then trained the network to predict the time evolution of initial condition as given in figure 2(a) and (b), respectively. The training loss after 1000 epochs (or iterations) reduced to an order of 1e-4 as given in figure 2(c) which indicates that the network can predict the highly non-linear phenomenon like shock.

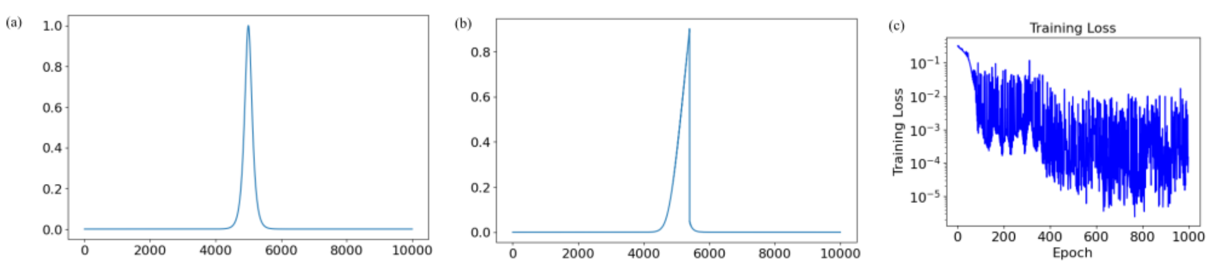

Figure 2: (a) Initial condition (b) Solution after t = 4s (c) training loss over epochs.

4 CONCLUSIONS

The Fourier neural network is capable for mapping between non-linear states rendering the network to be a good model for predicting the shock wave refraction over air-water interface. The operator network will be 1000 times faster than the conventional CFD methods in predicting the flow field. The results of the operator training and testing with the shock refraction over air-water will be presented in the full paper.

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