Bayesian inversion of piezometric and displacement data to characterize aquifer properties

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Introduction

The Alto Guadalentín Basin is one of the most important agricultural areas in Spain (Figure 1a). The exploitation of groundwater started in the 1960s to compensate for the scarcity of water and reached the maximum in the 1980s following the increasing anthropogenic activities. The over-exploitation of groundwater led to a significant decline of the piezometric level. Although new regulations have restricted the extraction of groundwater since 1989, these efforts had limited effects. The piezometric level in the vicinity of some active wells has declined up to 200 m from 1960 to 2012. As a direct consequence of such a large drawdown, the basin is experiencing one of the greatest rates of subsidence in Europe, with local rates exceeding 10 cm/yr. In the last decades, a series of studies have been conducted including geological surveys, subsidence monitoring, and deterministic numerical modeling (Cerón García 1995; Bonì et al. 2015; Ezquerro et al. 2017) providing investigations and explanations of the ongoing processes.

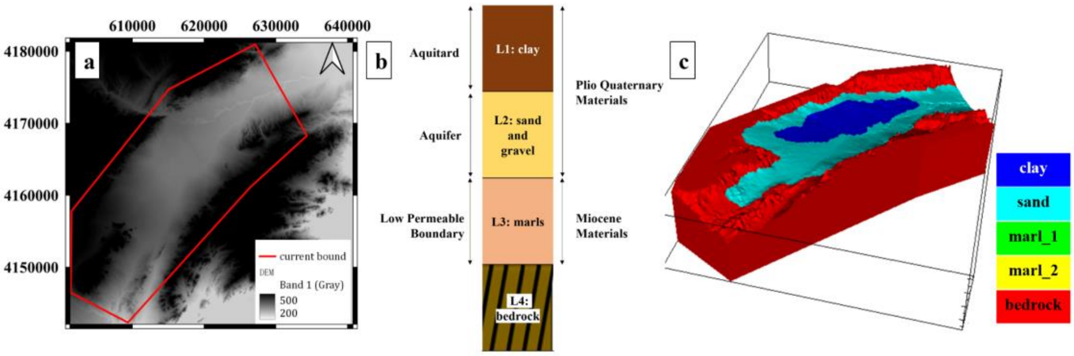

Figure 1 The Alto Guadalentín Basin. (a) Location of the aquifer system and digital elevation model showing the 2D model boundary, (b) Lithostratigraphic model of the aquifer system, and (c) 3D model domain with spatial distribution of the sedimentary bodies.

The aquifer system is composed of three main stratigraphic units (Figure 1b-c). The top clay layer is an aquitard but highly compressible, so this layer accounts for a large proportion of subsidence. The underlying sand and gravel layer is the main productive aquifer. The bottom of the aquifer system is mainly constituted by marls. There is some minor extraction within this layer even though it is much less permeable than the sand and gravel unit. However, it is challenging to perform history-matching, or even predict the aquifer system response to the human-induced stress. Indeed, from the hydraulic and geomechanical point of view, the behaviors of aquifer systems are governed by key parameters such as hydraulic conductivity and compressibility. On the other hand, these parameters are affected by high uncertainty, either because they are naturally heterogenous or because of inadequate observations. Overlooking these uncertainties may cause biased estimations of soil properties and compromised numerical results. Therefore, a Bayesian-based scheme is here implemented to reduce the uncertainty of hydraulic conductivity and compressibility by minimizing the residuals of the model outcomes with respect to the available measurements. This investigation focuses on two variables, namely the compressibility of the clay unit C_{mc} and the hydraulic conductivity of the sandy aquifer K_s, owing to their major contributions to land subsidence and groundwater production.

Methodology

The proposed framework intends to take advantage of several novel techniques including land surface deformation measurements by satellite remote sensing, an advanced numerical model that couples unsaturated groundwater flow with three-dimensional geomechanics, and the estimation of the system uncertainty by surrogate-based approaches. A 3D coupled variably-saturated groundwater flow-geomechanical model is applied to describe the interactions between hydrogeology and geomechanics (Nardean et al. 2021). This model allows to simulate how, in deformable porous media, the pore space in the reference bulk volume varies with the displacement of the solid grains and the pressure of the wetting phase. Then, a Bayesian-based method is employed to calibrate the parameters of interest by using piezometric records available over the period from 1960 to 1989 and displacement data obtained by Advanced Differential radar Interferometry (A-DInSAR) technique (Bonì et al. 2015) over the interval between 1992 and 2012.

The simulation is split into two periods for the purpose of model parameters estimation. First, the groundwater flow (GW) simulator is activated alone to compute the piezometric level from 1960 to 1989. In this time window, the calibration of K_s is conducted by using piezometric records from several monitoring wells. Displacement data are not available in this time frame. Then, the coupled model is run for the second time window (1992-2012) to simulate the piezometric and the displacement fields. Here, the compressibility C_{mc} is calibrated by assimilation of both land displacements and piezometric records.

In this framework, the model parameters are characterized by log-uniform prior probability density functions (pdfs) where $log(K_s/2) \sim U[-2, 0]$ m/d and $log(K_s/2) \sim U[-2, 0]$ KPa^{-1}). We initially tested the Markov Chain Monte Carlo (MCMC) method, a sampling algorithm, which calls for the simulation of the full problem multiple times. Since the computational cost associated with this strategy was unaffordable, we employed a surrogate-based approach where the full problem was replaced by a simpler, and much faster proxy to evaluate. In this work, the surrogate is based on the sparse grid approach given the tradeoff of accuracy and the training cost. This numerical technique is designed to compute the integral or interpolant of high dimensional functions from a set of collocation knots. Specifically, Leja knots are sampled from the parameter space to reach a fast convergence (Piazzola and Tamellini 2022). Besides, Leja knots have a nested structure where the knots from lower interpolation levels are a subset of higher levels, so the existing knots can be used in a step-by-step approach to gradually increase the accuracy of the surrogate. Prior to performing the Bayesian inversion by MCMC, the goodness of surrogate model is validated by evaluating the mean square error (MSE) indicator $E_{mse} = \sqrt{\frac{1}{I}\sum_{i=1,\cdots,I}^{I} \quad \frac{U_{\Lambda}(K_i)-U(K_i)}{U(K_i)}}$, where I is the number of validation points, U_{Λ} and U are solutions of the surrogate model and the numerical model respectively.

Preliminary result

GW outcome in the time window 1960-1989 results in a drawdown which mainly concentrates in the southeast and partially northwest portions of the basin. This pattern is consistent with the distribution of pumping wells. Along the vertical direction, the drawdown mainly occurs within the sand and gravel layer and marl layer. The maximum decline reaches more than 100 m over three decades where the saturation degree also experiences a pronounced reduction in the shallowest soil. The pressure head within the clay layer presents a lag response and ends up with a drop around 20 m. The computed drawdown reasonably matches the piezometric records at wells in the southwestern part of the basin but reaches a half of recorded values to the northeast. These differences should be shrunk after the parameter calibration.

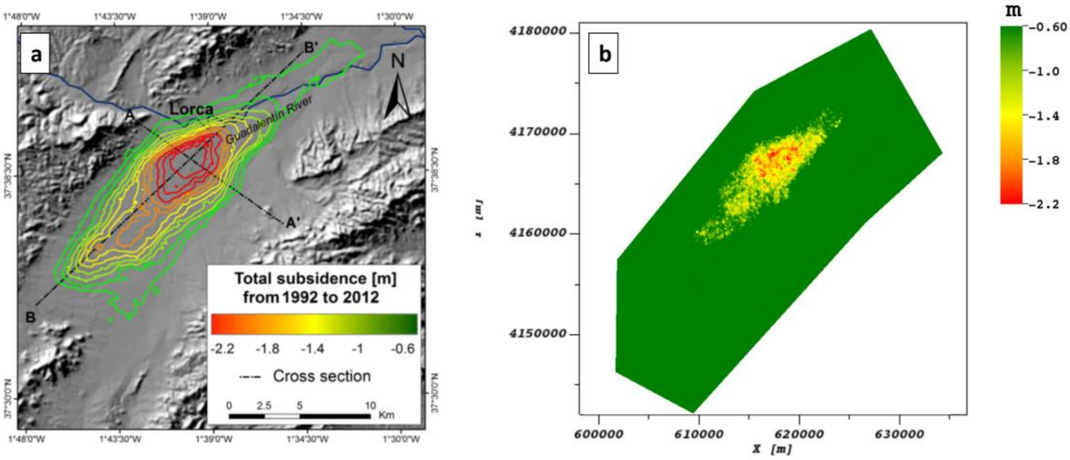

Figure 2 Land subsidence over the Alto Guadalentín Basin. (a) Total subsidence from 1992 to 2012 as obtained with the A-DInSAR technique (Bonì et al. 2015) (b) Model results from the 3D variably-saturated groundwater flow coupled with geomechanics in the time window between 1960 and 1992. Assuming K_S=0.2 m/d and C_{mc}=1e^{-4} .

The coupled model is also preliminary run over this time window to compute the amount of land displacements during the first 30 years of aquifer exploitation. The drawdown obtained by the coupled model has a similar pattern, but in the main productive area the piezometric decline is 10 m greater than the values obtained by the GW model when running alone. This difference is mainly caused by the variation of porosity, and consequently of aquifer storage, that is accounted for in the coupled formulation but neglected in GW simulations. In fact, aquifer storage is constant in the GW model while it varies in the coupled model decreasing as the aquifer compacts. The numerical results in terms of land subsidence agree with the spatial behavior of the A-DInSAR data. Although the simulation and A-DInSAR data span different time series, the shape and maximum value of the subsidence bowls look consistent (Figure 2). The coupled model also suggests the horizontal displacement is below 0.4 m.

The surrogate of the GW model for the first time window was obtained by testing an increasing number of knots and, thus, evaluating the accuracy of the surrogate solution. A final number of 9 knots are chosen as the MSE is on the order of centimeters. The response surfaces obtained with the surrogate for most of the monitoring wells are smooth and show the expected piezometric level decline over time, particularly over the years with a greater withdrawal (Figure 3). For the same year, the piezometric level is generally lower with a smaller K_S as expected (Figure 3a). The remaining response surfaces have an abnormal rising when K_S is close to the lower threshold, likely due to the narrower drawdown bowl that develops when the hydraulic conductivity decreases (Figure 3b). Undoubtedly, this surrogate model based on sparse grids provides highly reliable approximations of the numerical model.

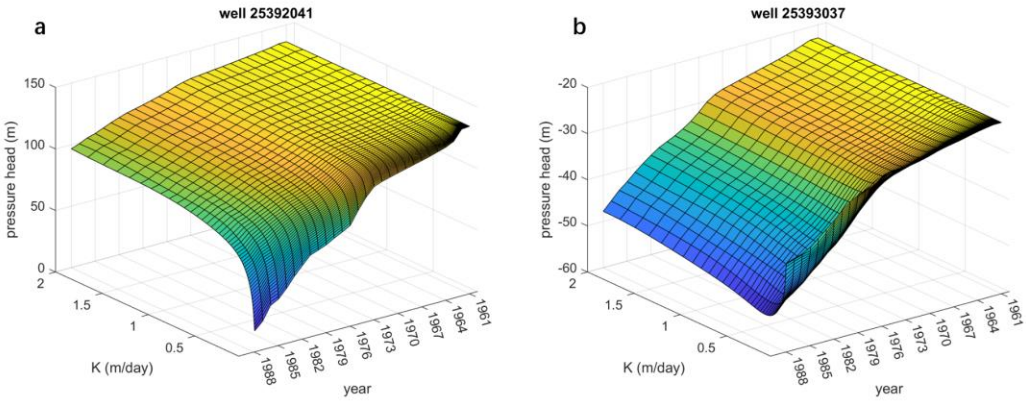

Figure 3 The response surfaces obtained by the surrogate model at two monitoring wells. (a) Well 25392041 represents the kind of wells which are in the main productive area, the pressure head is positively related to K_S (b) Well 25393037 is a bit far away from the productive wells. Wells like 25393037 are less affected by drawdown when K_S is low.

Conclusions

The proposed numerical model is capable of describing land subsidence caused by the groundwater withdrawal in a variably saturated aquifer system. In addition, the sparse grid approach used to reduce the computational burden presents a good ability on approximating the numerical results in terms of accuracy and cost. Inverse parameter estimation for K_S with the piezometric data for the first period is ongoing. Once Bayesian inversion will be validated with a proper posterior distribution of K_S, this study will move to the second period calibration by using the coupled model and A-DInSAR outcomes.

References

Bonì, Roberta, Gerardo Herrera, Claudia Meisina, Davide Notti, Marta Béjar-Pizarro, Francesco Zucca, Pablo J. González, et al. (2015). "Twenty-Year Advanced DInSAR Analysis of Severe Land Subsidence: The Alto Guadalentín Basin (Spain) Case Study." Engineering Geology 198 (November): 40–52. https://doi.org/10.1016/j.enggeo.2015.08.014.

Cerón García, Juan Carlos, (1995). "Estudio hidrogeoquímico del acuífero del alto guadalentín (Murcia)." https://digibug.ugr.es/handle/10481/55814.

Ezquerro, Pablo, Carolina Guardiola-Albert, Gerardo Herrera, José Antonio Fernández-Merodo, Marta Béjar-Pizarro, and Roberta Bonì, (2017). "Groundwater and Subsidence Modeling Combining Geological and Multi-Satellite SAR Data over the Alto Guadalentín Aquifer (SE Spain)." Geofluids 2017: 1–17. https://doi.org/10.1155/2017/1359325.

Nardean, S., M. Ferronato, Y. Zhang, S. Ye, X. Gong, and P. Teatini, (2021). "Understanding Ground Rupture Due to Groundwater Overpumping by a Large Lab Experiment and Advanced Numerical Modeling." Water Resources Research 57 (3). https://doi.org/10.1029/2020WR027553.

Piazzola, Chiara, & Lorenzo Tamellini, (2022). "The Sparse Grids Matlab Kit -- a Matlab Implementation of Sparse Grids for High-Dimensional Function Approximation and Uncertainty Quantification." arXiv. http://arxiv.org/abs/2203.09314.