Convergence of Monte Carlo methods for neutron noise

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Outline

1. Modelling noise
2. Monte-Carlo for neutron noise
3. Theoretical framework
4. Application: infinite homogeneous media
5. Discussion

1. Modelling noise

What is neutron noise ?

Definition

- **Small periodic variations** $\delta\varphi$ of the neutron flux around the stationary state φ_c
- A.k.a. **power reactor noise**

Causes

- **Periodic** perturbation
- Coolant density fluctuations
- Vibrations of the mechanical structures of the core (vessel, assemblies, fuel pin, …)
- Etc.

Applications

- Non-invasive core monitoring
- Measurement of the coolant properties
- Detection of abnormal vibrations

Modelling neutron noise sources

- Boltzmann equation :
  - Variables : $\vec{r}, E, \vec{\Omega}, t$
  - Unknown : φ
  - Parameters : Σ
- Noise source : **cross-sections** perturbations
  - $\Sigma(\mathrm{t}) = \langle\Sigma\rangle + \delta\Sigma(t)$
  - Small periodical perturbation $\delta\Sigma$

Examples

Variable strength absorber

- $\Sigma_a(t) = \Sigma_a(1 + \epsilon_a \cos \omega t)$

Density oscillation

- $N(t) = N(1 + \epsilon \cos \omega t)$
- $\Sigma_k(t) = \sigma_k N(t) = \Sigma_k(1 + \epsilon \cos \omega t)$

Mechanical vibration

- Materials "left" L and "right" R
- Moving interface $x_i(t) = x_0 + \epsilon \cos \omega t$
- $\Sigma(x,t) = \begin{cases} \Sigma^L \text{ if } x \leq x_i(t) \\ \Sigma^R \text{ if } x > x_i(t) \end{cases} = \langle\Sigma\rangle(x) + \sum c_n(x) e^{in\omega t}$

Derivation of the noise equations

- Boltzmann equation: $\mathfrak{B}\varphi(t) = 0$
- Operators: $\mathfrak{B} = \mathfrak{B}_0 + \delta\mathfrak{B}$
  - Mean values $\mathfrak{B}_0 \leftrightarrow \langle\Sigma\rangle$
  - Perturbations $\delta\mathfrak{B} \leftrightarrow \delta\Sigma$
- Criticality, for some k_{eff}: $\mathfrak{B}_0\varphi_c = 0$
- Flux: $\varphi(t) = \varphi_c + \delta\varphi(t)$

$$\underbrace{\mathfrak{B}_0\varphi_c}_{=0} + \delta\mathfrak{B}\varphi_c + \mathfrak{B}_0\delta\varphi + \underbrace{\delta\mathfrak{B}\delta\varphi}_{\substack{\text{Neglected}\\\text{(othodox linearization)}}} = 0$$

- From time to frequency: Fourier transform $\hat{f}(\omega) = \int_{-\infty}^{+\infty} f(t)e^{-i\omega t}dt$

- Linearized noise equation in the frequency domain :

$$\widehat{\mathfrak{B}_0}\widehat{\delta\varphi}(\omega) = -\widehat{\delta\mathfrak{B}}\varphi_c$$

- Angular frequency ω is now a parameter, i.e. this can be solved independently for each ω
- The equation has the form of a source problem :
  - Right-hand side, “noise source”
  - Left-hand side, modified transport operator acting on the unknown $\widehat{\delta\varphi}$
- **Complex-valued equation**

$$\widehat{\mathfrak{B}_0} = \left(\frac{\boldsymbol{i\omega}}{\boldsymbol{v}} + \vec{\Omega}\cdot\nabla + \Sigma_t\right) - \iint f_s\Sigma_s dE'd\vec{\Omega}' - \frac{\chi_p}{4\pi k_{\text{eff}}}\iint \nu_p\Sigma_f dE'd\vec{\Omega}' - \sum_j \frac{\chi_d^j}{4\pi k_{\text{eff}}}\frac{\boldsymbol{\lambda_j}}{\boldsymbol{\lambda_j + i\omega}}\iint \nu_d^j\Sigma_f dE'd\vec{\Omega}'$$

2. Monte Carlo for neutron noise

Monte Carlo for neutronics

Monte Carlo for neutronics

Pros & Cons

- **Reference** computation : transport equation, exact geometry, continuous energy
- Good parallelization
- High computing time

Method

- Sample neutrons from **source**
- **Flight** : Sample a flight distance to next collision and move particle
- **Collision** : Sample collision type, outgoing multiplicity, direction and energy
- **Record** results

A trick : importance sampling

- Give each particle a statistical weight w
- You can freely change the probability p of an outcome, as long as you also change the particle's weight according to : $p'w' = pw$.

Another trick : Russian roulette

- Consider a particle with weight w. You can kill it with probability $(1 - w)$ or let it survive with probability w and weight 1.
- Avoids low contribution particle

Monte Carlo for neutron noise

Monte-Carlo for neutron noise

- Complex weights $w \in \mathbb{C}$
- Delayed fission: $w' = \frac{\lambda}{\lambda + i\omega} w$
- Two different methods for $\frac{i\omega}{v}$

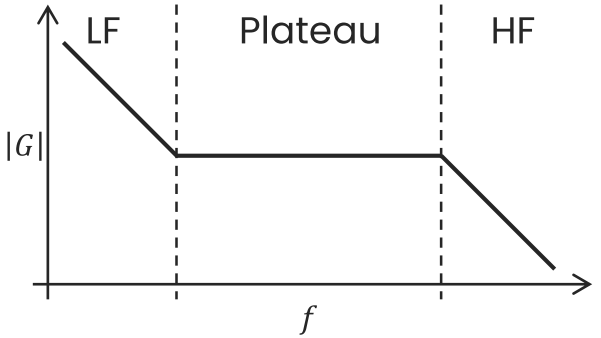
Flight-based method (Yamamoto)

- Complex total cross-section $\Sigma_t + \frac{i\omega}{v}$
- Length s flights: probability density $e^{-\Sigma_t s}$; weight $w' = we^{-\frac{i\omega}{v}s}$

Collision-based method (Rouchon et al.)

- Real total cross-section $\widetilde{\Sigma_t} = \Sigma_t + \frac{\omega}{v}$
- New « copy » reaction: real cross-section $\Sigma_c = \frac{\omega}{v}$; same $E, \vec{\Omega}$; weight $w' = (1 - i)w$

- Provides unbiased estimator of the neutron noise.
- Some **convergence issues**, especially at low and high frequency. In particular, it can create diverging populations of particle having weights of opposite sign.
- **Weight cancellation**: cancel out opposite sign particles. Efficient for convergence and variance reduction.



Fundamental transfert function

3. Theoretical framework

Complex valued Monte Carlo Games

Standard Monte Carlo

- Fredholm's integral equation of the second kind:

$$\varphi(x) = \int K(x,y)\varphi(y)dy + q(x)$$

- Compute a response:

$$R = \int h(x)\varphi(x)dx$$

- Requirement: real, non-negative kernel and source.

Suitable for Monte Carlo

Noise equation

- Integro-differential
- Complex-valued

Integral equation

- Fredholm's
- Complex kernel and source

Modified equation

- Fredholm's
- Same response
- Real non-negative kernel and source
- Dimension +1 = "Phase"

Transformation

- Choice of kernels Z_μ over the extra dimension.

Complex weighted Monte Carlo

- Equivalent to standard Monte Carlo applied to a modified equation, for some choice of Z_μ kernels.

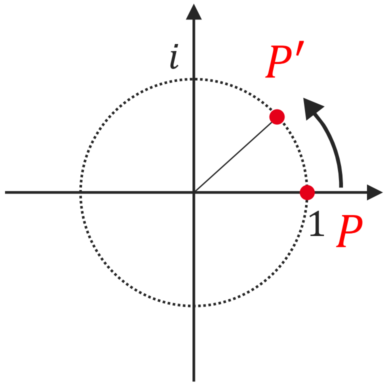
Kernels of the modified equation

- Kernel Z_μ represents multiplication by $\mu \in \mathbb{U}$: $\forall \theta', \int Z_\mu(\theta', \theta)\theta d\theta = \mu\theta'$
- Different possible choices

2π-System

- Additional dimension $\theta \in \mathbb{U}$
- Upon multiplication by μ, rotation:

$$\begin{cases} \theta' = \mu\theta \\ w' = w \end{cases}$$



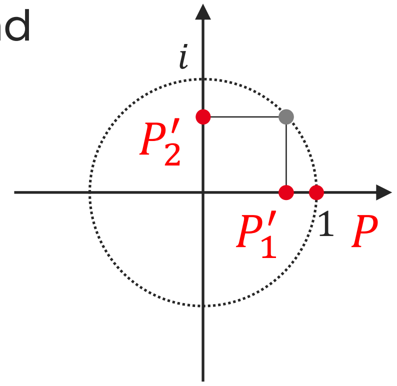
- Equivalent to using complex weights.

4-System

- Additional dimension $\theta \in \{+1, -1, +i, -i\}$
- Upon multiplication by μ, particle split in real and imaginary part:

$$\begin{cases} \theta'_1 = \pm 1 \\ w'_1 = |\Re(\mu\theta)| w \end{cases}$$

$$\begin{cases} \theta'_2 = \pm i \\ w'_2 = |\Im(\mu\theta)| w \end{cases}$$



- Equivalent to using 4 particle "species".

Introducing noise k eigenvalue

Neutronics

- Dominant eigenvalue: effective multiplication factor k_{eff}
- Fixed source calculation converges if and only if $k_{\mathrm{eff}} < 1$

Noise

- Fixed source problem

Physical equation

- « Physical » eigenvalue k

Modified equation

- Eigenvalue $\tilde{k}$

- We show that $|k| \leq |\tilde{k}|$

Weight cancellation

- Linear model of weight cancellation (H. Belanger)
- Supress all non-physical eigenvalues, so that $\tilde{k} = k$

3 Regimes

- $|k| \leq |\tilde{k}| < 1$:
  Convergent
- $|k| < 1 \leq |\tilde{k}|$:
  Conditionally convergent
  (diverges, unless weight
  cancellation is enforced)
- $1 < |k| \leq |\tilde{k}|$:
  Divergent
  (even with weight
  cancellation)

4. Application: infinite homogeneous media

Infinite homogeneous media

Description

Homogeneous, infinite, one energy group, one precursor group

Methodology

Analytical formula for $\tilde{k}$ → Monte Carlo computation of $\tilde{k}$ → Monte Carlo convergence verification

Result

All results confirmed ✓

Variants

Frequency
- Large range, from low frequencies to high frequencies

Methods
- Flight-based
- Collision-based

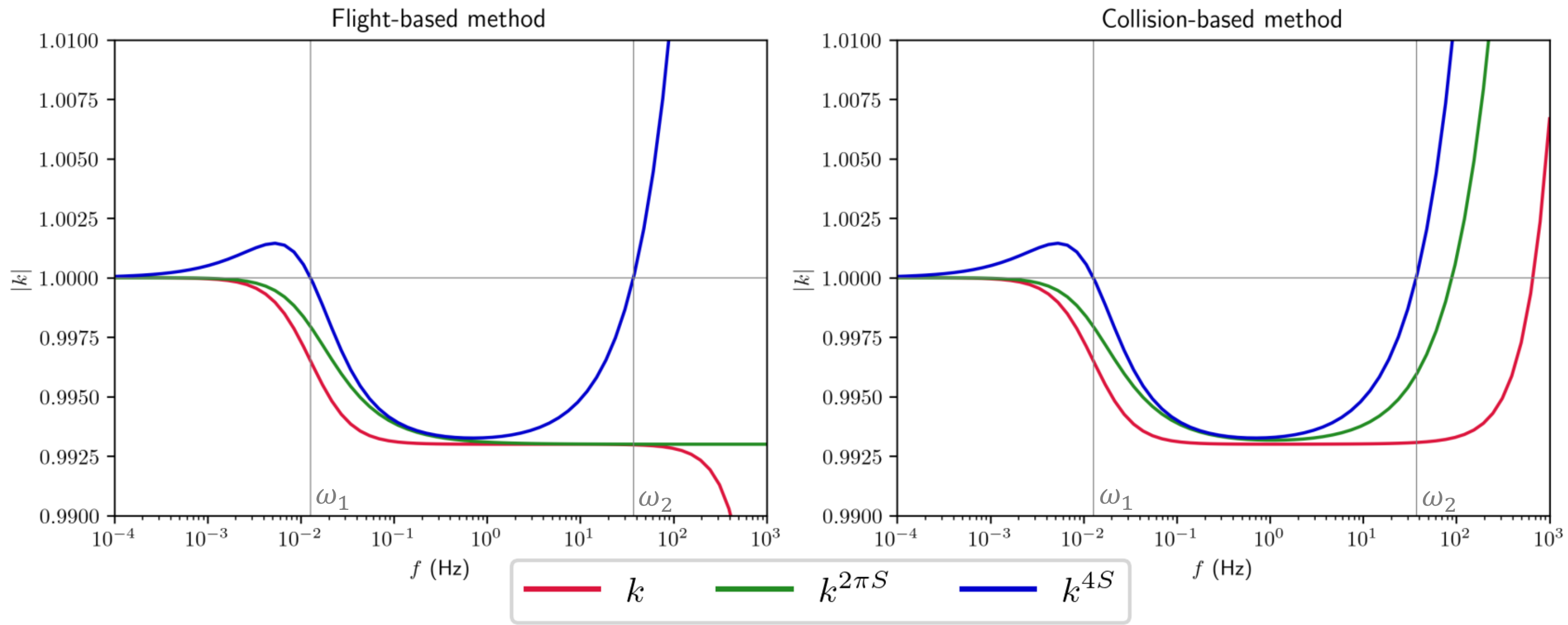
Rules
- Analog
- Non-analog : implicite capture, russian roulette and forced fission

Species
- $2\pi S$: complex weights
- $4S$: Four species $1, -1, i, -i$

Russian Roulette
- « Square » : $\Re w$ & $\Im w$
- « Norm » : $|w|$

Weight Cancellation
- Without
- Simple *binning procedure*

Model results



		Low frequency	Plateau	High frequency			
			ω_1	ω_2	ω_{cb1}	ω_{cb2}	ω_{fb}
Flight-based	$2\pi S$	Convergent					
	$4S$	Conditionally Convergent	Convergent	Conditionally Convergent			$+\infty$
Collision-based	$2\pi S$	Convergent			C.C.	Divergent	
	$4S$	Conditionally Convergent	Convergent	C. C.		Divergent	

A note

During question session, the reason for the difference of "physical" eigenvalue between the two methods has been raised. This is due to a choice of slightly different definition for $\tilde{k}$ in the two methods, that allows to encompass all relevant information in the $\tilde{k}$ eigenvalue.

A more complete approach, beyond the scope of this presentation, has been developed and will be published in the future.

5. Discussion

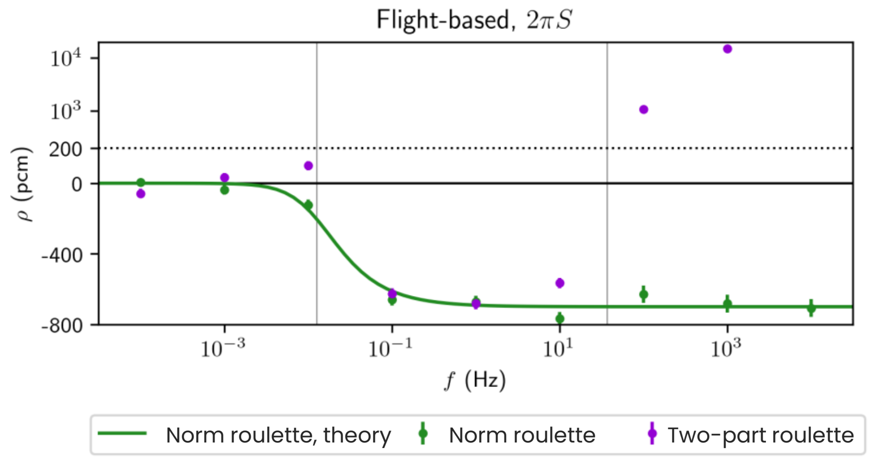
Discussion

Results

- Reproduce well the convergence properties from the literature
- Discrepancies studied and explained

Russian Roulette

- Previously, two-part Russian roulette ($\Re w$ & $\Im w$): increases the $\tilde{k}$ eigenvalues.
- New norm-based Russian roulette ($|w|$): do not increase $\tilde{k}$ eigenvalues, recommended.



Conclusions

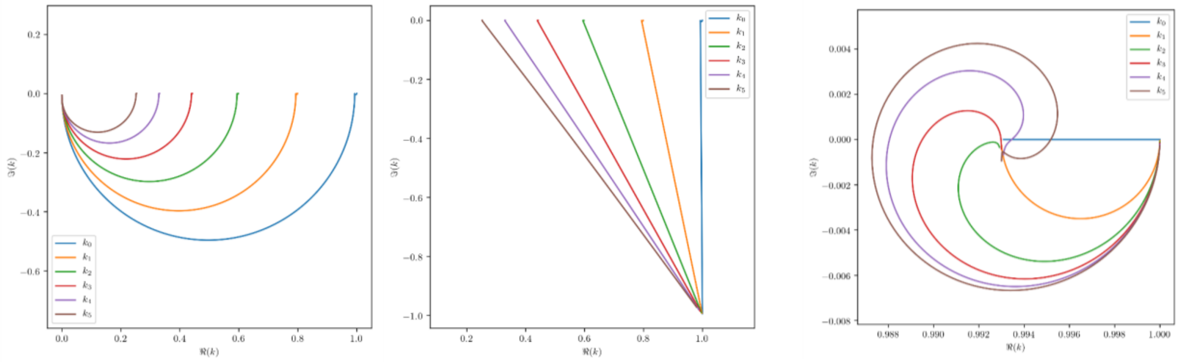
Results

- Draft theory of Complex Monte Carlo
- Benchmark: confirmed theory and reproduced results from literature

Paper at SNA+MC 2024 (10/2024)

Ongoing work

- Refined theory
- Benchmark: Rod model
  - Spatial effects
  - Richer model but same conclusions so far



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Merci
Thank you

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